Exercise 6

Find an equation of the tangent line to the curve at the given point.

$$y = x^3 - 3x + 1, \quad (2, 3)$$

Solution

Start by finding the slope of the tangent line to the curve at $x = 2$.

$$m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{(x^3 - 3x + 1) - [(2)^3 - 3(2) + 1]}{x - 2}$$

$$= \lim_{x \to 2} \frac{(x^3 - 3x + 1) - (3)}{x - 2}$$

$$= \lim_{x \to 2} \frac{x^3 - 3x - 2}{x - 2}$$

$$= \lim_{x \to 2} \frac{(x - 2)(x + 1)^2}{x - 2}$$

$$= \lim_{x \to 2} (x + 1)^2$$

$$= (2 + 1)^2$$

$$= 9$$

The general equation of a line is

$$y = mx + b.$$

Here the slope is $m = 9$.

$$y = 9x + b$$

Use the fact that the line passes through $(2, 3)$ to determine b.

$$3 = 9(2) + b$$

$$3 = 18 + b$$

$$b = -15$$

Therefore,

$$y = 9x - 15.$$

Below is a plot of the curve and the tangent line at $x = 2$.

$$y = x^3 - 3x + 1$$

$$y = 9x - 15$$